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Confidence intervals for Expected Moments Algorithm flood quantile estimates

Timothy A. Cohn
U.S. Geological Survey, Reston, Virginia

William L. Lane
Golden, Colorado

Jery R. Stedinger
Department of Civil and Environmental Engineering, Cornell University, Ithaca, New York

Abstract. Historical and paleoflood information can substantially improve flood frequency estimates if appropriate statistical procedures are properly applied. However, the Federal guidelines for flood frequency analysis, set forth in Bulletin 17B, rely on an inefficient "weighting" procedure that fails to take advantage of historical and paleoflood information. This has led researchers to propose several more efficient alternatives including the Expected Moments Algorithm (EMA), which is attractive because it retains Bulletin 17B's statistical structure (method of moments with the Log Pearson Type 3 distribution) and thus can be easily integrated into flood analyses employing the rest of the Bulletin 17B approach. The practical utility of EMA, however, has been limited because no closed-form method has been available for quantifying the uncertainty of EMA-based flood quantile estimates. This paper addresses that concern by providing analytical expressions for the asymptotic variance of EMA flood-quantile estimators and confidence intervals for flood quantile estimates. Monte Carlo simulations demonstrate the properties of such confidence intervals for sites where a 25- to 100-year streamgage record is augmented by 50 to 150 years of historical information. The experiments show that the confidence intervals, though not exact, should be acceptable for most purposes.

1. Introduction

Researchers, consultants, and water managers in the private sector as well as state, local, and federal agencies have long been aware of the importance of historical flood information when estimating flood quantiles. Potential sources of information about past floods include geological evidence from slackwater deposits and botanical evidence preserved in flood-caused scars in trees, in addition to the traditional newspaper accounts and flood lines painted on buildings [*Stedinger and Baker*, 1987, and references therein].

This abundance of historical flood information has, in turn, motivated scientists and engineers to develop statistical methods that can exploit such nonstandard data to improve flood-frequency estimates [*Leese*, 1973; *Tasker and Thomas*, 1978; *Condie and Lee*, 1982; *Condie and Pilon*, 1983; *Condie*, 1986; *Stedinger and Cohn*, 1986; *Hosking and Wallis*, 1986a, 1986b; *Cohn and Stedinger*, 1987; *Lane*, 1987; *Stedinger and Cohn*, 1987; *Hirsch and Stedinger*, 1987; *Jin and Stedinger*, 1989; *Wang*, 1990a, 1990b; *Guo and Cunnane*, 1991; *Kuczera*, 1992; *Pilon and Adamowski*, 1993; *Frances and Salas*, 1994; *Kroll and Stedinger*, 1996; *England*, 1998]. Researchers have also attempted to quantify the information content of historical flood data and have generally concluded that it is surprisingly valuable. For example, accurate information about the significant floods that occurred during the past 200 years (many communities have such records) can, in theory, provide as much information about the magnitude of the 100-year flood as can be extracted from 150 years of continuous gage record [*Stedinger and Cohn*, 1987].

The value of historical information, however, depends on the statistical method used to compute flood-frequency estimates. In particular, the currently accepted flood-frequency methodology in the United States, described in Bulletin 17B [*Interagency Committee on Water Data* (*IACWD*), 1982], has been shown to be relatively inefficient when used with historical information [*Condie and Lee*, 1982; *Stedinger and Cohn*, 1986; *Lane*, 1987].

Several efficient methods for dealing with historical information have been proposed, such as maximum likelihood, probability-weighted moments, and order statistics [*Leese*, 1973; *Stedinger and Cohn*, 1987; *Wang*, 1990a, 1990b; *Durrans*, 1996; *Durrans et al*., 1999; *Koutrouvelis and Canavos*, 1999, 2000]. Most, however, would require modifying or abandoning the method of moments, Bulletin 17B's basic statistical structure. Such a change could create substantial practical and legal difficulties.

Recently, Lane and Cohn [*Cohn et al*., 1997; W. L. Lane, Method of moments approach to historical data, handout, 1995] proposed a less extensive modification to Bulletin 17B, the Expected Moments Algorithm (EMA). EMA employs Bulletin 17B's methodology and assumptions, even (for the most part) with respect to historical information. However, EMA substantially improves upon Bulletin 17B's efficiency: EMA's

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efficiency approaches the asymptotic bound for any estimator using historical flood information [*Cohn et al.*, 1997].

This paper fills a void in the work of *Cohn et al.* [1997] by providing methods to assess the uncertainty in EMA flood-quantile estimates. The paper also provides analytical formulas for the asymptotic variance of EMA moments and quantile estimators and shows how these can be used to compute approximate confidence intervals for EMA quantile estimates. Monte Carlo experiments illustrate the performance of the confidence intervals with data that an investigator might encounter in practice.

2. Statistical Model for Floods

As in Bulletin 17B [*IACWD*, 1982], we assume that $\{Q_1 \ldots Q_N\}$ are independent and identically distributed log-Pearson type 3 (LP-3) variates. Thus $X_i \equiv \ln(Q_i)$ has a Pearson type 3 (P-3) distribution with probability density function [*Lall and Beard*, 1982]:

$$f_\theta(x) = \begin{cases} \dfrac{\left(\dfrac{x-\tau}{\beta}\right)^{\alpha-1} \cdot \exp\left(-\dfrac{x-\tau}{\beta}\right)}{|\beta|\Gamma(\alpha)} & \text{for } \left(\dfrac{x-\tau}{\beta}\right) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $\boldsymbol{\theta} \equiv \{\alpha, \beta, \tau\}'$ is a vector of parameters and $\Gamma(\,)$ is the Gamma function [*Abramowitz and Stegun*, 1964].

The P-3 distribution can be characterized in terms of its first three noncentral moments (*Bobée* [1975] discusses estimation with the LP-3 moments), which are defined as

$$\boldsymbol{\mu}_{\mathbf{X}} \equiv \begin{bmatrix} \mu_1' \\ \mu_2' \\ \mu_3' \end{bmatrix} \equiv \begin{bmatrix} E_\theta[X] \\ E_\theta[X^2] \\ E_\theta[X^3] \end{bmatrix}. \tag{2}$$

Here the accents (e.g., μ_1') by convention indicate that the preceding scalar quantities are noncentral moments. However, to avoid confusion the accents are omitted from vectors of noncentral moments (e.g., $\boldsymbol{\mu}_{\mathbf{X}}$) because, by convention, for vectors an accent indicates transposition.

$E_{\boldsymbol{\theta}}[X^k]$ can be expressed as a binomial expansion:

$$E_\theta[X^p] = \sum_{j=0}^{p} \binom{p}{j} \beta^j \tau^{p-j} \left(\frac{\Gamma(\alpha+j)}{\Gamma(\alpha)}\right). \tag{3}$$

The relationship between the parameter $\boldsymbol{\theta}$ and the noncentral moments can be expressed in the following way:

$$\boldsymbol{\theta} = \boldsymbol{\theta}[\boldsymbol{\mu}] = \begin{bmatrix} \dfrac{4(\mu_2' - \mu_1'^2)^3}{(\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3)^2} \\ \dfrac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{2(\mu_2' - \mu_1'^2)} \\ \dfrac{\mu_3'\mu_1' - 2\mu_2'^2 + \mu_2'\mu_1'^2}{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3} \end{bmatrix} \tag{4}$$

$$\boldsymbol{\mu}_{\mathbf{X}} = \boldsymbol{\mu}[\boldsymbol{\theta}] = \begin{bmatrix} \alpha\beta + \tau \\ \alpha(1+\alpha)\beta^2 + 2\alpha\beta\tau + \tau^2 \\ \alpha(1+\alpha)(2+\alpha)\beta^3 + 3\alpha(1+\alpha)\beta^2\tau + 3\alpha\beta\tau^2 + \tau^3 \end{bmatrix}. \tag{5}$$

3. Estimating Sample Moments and Quantiles

Noncentral sample moments for ordinary (uncensored) data can be computed directly and simply:

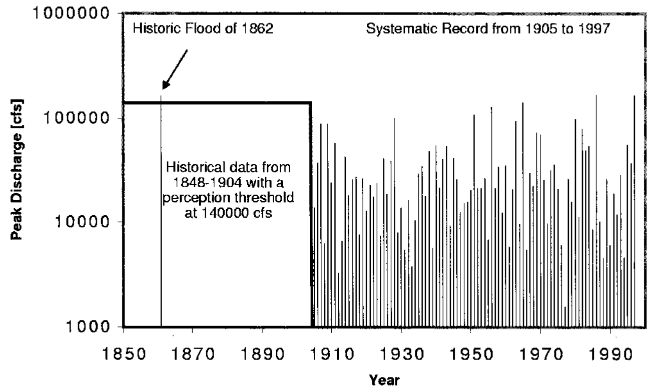

Figure 1. A flood record for American River at Fair Oaks combining systematic data from 1905 to 1997 with historical information from 1850 to 1904. Note that one "historic flood" occurred in 1862; the other 54 annual peak flows between 1850 and 1904 were below 140,000 (cfs) and are thus treated as censored data.

$$\hat{\boldsymbol{\mu}} = \hat{\mathbf{M}} = (1/N) \sum_{i=1}^{N} \mathbf{X}_i \equiv (1/N) \sum_{i=1}^{N} \begin{bmatrix} X_i \\ X_i^2 \\ X_i^3 \end{bmatrix}. \tag{6}$$

Here N denotes the sample size, bold letters indicate vectors or matrices, and carets are used to identify estimators.

3.1. Censored Data

Historical and paleoflood information can often be characterized statistically as "Type I censored data" [*Leese*, 1973; *Stedinger and Cohn*, 1986]. The term censored refers to the situation in which the values of some of the observations in a sample are unknown [*Schneider*, 1986]. Censoring occurs for various reasons.

3.1.1. Perception threshold: We have a record of the value of X only when the magnitude of X exceeded some perception threshold [*Stedinger and Cohn*, 1986; *Potter*, 1999].

3.1.2. Instrument range: For economic or technological reasons we may choose to employ instruments (or experimental designs) that record only a limited range of a geophysical phenomenon, and values outside the range will be reported as "out-of-range" or "censored" data. For example, extreme groundshaking can exceed the range of older seismometers [*Benz et al.*, 2000].

3.1.3. Deliberate: We may choose to censor data after the fact. For example, deliberately censoring data below a threshold can be a valid and effective way to make a model insensitive to low outliers that might otherwise overly influence model "fit" (see *Potter* [1999] for discussion).

Figure 1 depicts an annual peak flood series for the American River (California) from 1850 to 1997. The series is "systematic" for the $N_S = 93$ most recent years; we know the value of each annual peak flood because there was a streamgage at the site. However, we also know something about each of the $N_H = 55$ annual peak floods that occurred between 1850 and 1904. We know the magnitude of the "historic flood" of 1862 because the flood was so big that people recorded its magnitude at the time; we can also reconstruct its magnitude from the geological evidence that it left behind. Likewise, we know that the 54 other annual peak flows that occurred during the "historical period" did not leave evidence behind. Their magnitudes were below the perception threshold (here drawn at 140,000 cfs) because otherwise we would have records, as we do for the 1862 event.

One consequence of censoring is that instead of working with X directly, one is faced with a noninvertible function of the data, $\psi(X)$:

$$\psi(X) = \begin{cases} \text{“less than } a\text{”} & X < a \\ X & a \le X \le b \\ \text{“greater than } b\text{”} & X > b \end{cases} \tag{7}$$

One knows which category X is in: less than the perception threshold a; within the closed interval $[a, b]$; or greater than b. Also, the magnitude of X is known if X is within $[a, b]$. However, only a bound on X can be identified if $X < a$ or $X > b$.

This kind of censoring results in three categories of data (“less,” “between,” and “greater”), and the number of observations in each of these categories is a random variable, denoted N_L, N_B, and N_G, respectively. Because each X must fall into one of the three categories, the total sample size is constant: $N = N_L + N_B + N_G$.

Recording censored data requires some extra bookkeeping, but there is a simple way to do this: Each observation, say x, can be recorded as a vector, (t_L, t_U), where t_L is a lower bound on the value of the observation and t_U is a corresponding upper bound. How (t_L, t_U) represents what we know about x depends on the value of x:

Value of x	Category	(t_L, t_U)
$x < a$	L	$(-\infty, a)$
$a \le x \le b$	B	(x, x)
$x > b$	G	(b, ∞)

3.2. Estimating Moments From Censored Data

There is no simple method comparable to (6) for computing the sample moments of $\{X_1 \ldots X_N\}$ from the observables $\{\psi(X_1) \ldots \psi(X_N)\}$. Statisticians often employ maximum likelihood estimators (MLE) to utilize categorical data because of their many desirable properties [*Kendall et al.*, 1979; *David*, 1981; *Stedinger and Cohn*, 1986; *Cohn*, 1988; *Cohn et al.*, 1992]. Given regularity conditions, one can show that MLEs are asymptotically efficient and unbiased [*Kendall et al.*, 1979]. However, they are not easy to use or to explain, and MLEs are particularly troublesome when applied to the P-3 distribution because the P-3 does not satisfy the regularity requirements (see *Hirose* [1995] for discussion).

There is also a nontechnical issue to consider. As a practical matter, adoption of MLEs for flood-frequency analyses in the United States would involve a substantial shift away from the current method of moments approach in the Bulletin 17B guidelines for Federal agencies, likely affecting floodplain delineations across the nation. From a regulatory perspective, such a change would create many complications and be difficult to implement. As a consequence, for the near future it seems likely that the method-of-moments/LP-3 framework described in Bulletin 17B will continue to be recommended for most flood-frequency studies in the United States. Under these circumstances, EMA has great appeal [*England*, 1998; *Potter*, 1999].

3.3. EMA: What It Is And How It Works

EMA is an iterative procedure for locating a fixed point, $\hat{\mathbf{M}} \equiv [\hat{m}_1, \hat{m}_2, \hat{m}_3]'$, that solves the nonlinear equation:

$$\hat{\mathbf{M}} = (1/N) \sum_{i=1}^{N} \boldsymbol{\chi}(\psi(X_i), \hat{\mathbf{M}}) \mathcal{I}[\psi(X_i)], \tag{8}$$

where

$$\mathcal{I}[X] \equiv \begin{bmatrix} \mathcal{I}(X < a) \\ \mathcal{I}(a \le X \le b) \\ \mathcal{I}(X > b) \end{bmatrix} \tag{9}$$

$$\mathcal{I}(\text{condition}) \equiv \begin{cases} 1 & \text{condition = true} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$\boldsymbol{\chi}(\psi(X), \mathbf{M}) = \begin{bmatrix} E_{\boldsymbol{\theta}[\mathbf{M}]}[X|X < a] & X_i & E_{\boldsymbol{\theta}[\mathbf{M}]}[X|X > b] \\ E_{\boldsymbol{\theta}[\mathbf{M}]}[X^2|X < a] & X_i^2 & E_{\boldsymbol{\theta}[\mathbf{M}]}[X^2|X > b] \\ E_{\boldsymbol{\theta}[\mathbf{M}]}[X^3|X < a] & X_i^3 & E_{\boldsymbol{\theta}[\mathbf{M}]}[X^3|X > b] \end{bmatrix} \tag{11}$$

$$E_{\boldsymbol{\theta}[\mathbf{M}]}[X^p|X < a] = E_{\boldsymbol{\theta}}[X^p| -\infty < X < a] \tag{12}$$

$$E_{\boldsymbol{\theta}[\mathbf{M}]}[X^p|X > b] = E_{\boldsymbol{\theta}}[X^p|b < X < \infty] \tag{13}$$

$$E_{\boldsymbol{\theta}}[X^p|a \le X \le b] = \frac{\int_a^b x^p f_\theta(x)\, dx}{\int_a^b f_\theta(x)\, dx} \tag{14}$$

$$E_{\boldsymbol{\theta}}[X^p|a \le X \le b] = \begin{cases} \sum_{j=0}^{p} \binom{p}{j} \beta^j \tau^{p-j} \left(\dfrac{\Gamma\left(\alpha + j, \dfrac{b-\tau}{\beta}, \dfrac{a-\tau}{\beta}\right)}{\Gamma\left(\alpha, \dfrac{b-\tau}{\beta}, \dfrac{a-\tau}{\beta}\right)} \right) & \beta < 0 \\ \sum_{j=0}^{p} \binom{p}{j} \beta^j \tau^{p-j} \left(\dfrac{\Gamma\left(\alpha + j, \dfrac{a-\tau}{\beta}, \dfrac{b-\tau}{\beta}\right)}{\Gamma\left(\alpha, \dfrac{a-\tau}{\beta}, \dfrac{b-\tau}{\beta}\right)} \right) & \beta > 0 \end{cases} \tag{15}$$

$$\Gamma(\alpha, a, b) = \int_{\max 0,a}^{\max 0,b} t^{\alpha-1} \exp(-t)\, dt. \tag{16}$$

Starting with an initial estimate $\mathbf{M} = \mathbf{M}_0$ inserted into the right-hand side of (8), a value $\mathbf{M}_1$ is obtained on the left. This value is then inserted into the right-hand side, and in this way, EMA iterates until a fixed point, $\hat{\mathbf{M}}$, is located. EMA has been found to converge reliably [*Cohn et al.*, 1997].

3.4. Asymptotic Variance of EMA Moments Estimator

The formula for the asymptotic variance of the EMA moments estimator, denoted $\tilde{\Sigma}$, is derived in Appendix A (section A1). It is obtained by linearizing the expectations in (8) and solving for $\mathbf{M}$ in terms of the sample X_i values. The estimator $\tilde{\Sigma}$ is then expressed as a function of the population parameters, the record lengths, and the censoring thresholds.

Equation (56) for $\tilde{\Sigma}$ in section A1 can serve two purposes: (1) As an approximation to the true variance-covariance matrix of the non-central moments given the true parameters of the distribution. This is denoted $\tilde{\Sigma}$; and (2) As an estimator of the variance-covariance matrix given estimated parameters. This is denoted $\hat{\Sigma}$.

3.5. Multiple Censoring Thresholds

Section 3.1 of this paper addresses the case where all observations, $[X_1 \ldots X_N]'$ are subject to the same censoring thresholds. Although one can imagine censored data as described in section 3.1 (crest stage gages report exact stage information only for floods that rise above the bottom of the gage and do not exceed the top [*Friday*, 1965]) the typical situation is more complicated. It includes multiple classes of censored data, possibly a systematic (uncensored) record combined with historical or paleoflood data covering a longer period. Censoring thresholds might vary over time [e.g., *Stedinger et al.*, 1988].

To accommodate multiple thresholds one replaces the matrices **B**, **C**, **D** in section A1 by $\Sigma\, \mathbf{B}_j$, $\Sigma\, \mathbf{C}_j$, and $\Sigma\, \mathbf{D}_j$, where the subscripts refer to classes of censored data. The total record length N is simply the sum of the record lengths for each class. The rest of the matrix algebra remains essentially unchanged.

3.6. EMA Quantile Estimator

The quantile function for the P-3 distribution is

$$X_p = F_{\boldsymbol{\theta}}^{-1}(p) = \begin{cases} \tau + \beta\Gamma^{-1}(\alpha, p) & \beta > 0, \\ \tau + \beta\Gamma^{-1}(\alpha, 1-p) & \beta < 0, \\ \tau & \beta = 0. \end{cases} \quad (17)$$

F^{-1} is the inverse of the P-3 cumulative distribution function, p is the nonexceedance probability, and Γ^{-1} is the inverse of the incomplete regularized Gamma function [*Abramowitz and Stegun*, 1964, pp. 255–266]. An estimator for P-3 quantiles is given by substituting estimated moments into (4) to obtain estimated parameters $\hat{\boldsymbol{\theta}} = \{\hat{\tau}, \hat{\alpha}, \hat{\beta}\}'$, which are then used to obtain the quantile estimator:

$$\hat{X}_p \equiv F_{\hat{\boldsymbol{\theta}}}^{-1}(p). \quad (18)$$

4. Confidence Intervals

Accurate confidence intervals for P-3 quantiles (LP-3 confidence intervals are obtained by exponentiating the endpoints of the P-3 intervals) have been developed and extensively tested for use with complete data [*Chowdhury and Stedinger*, 1991; *Hu*, 1987; *Whitley and Hromadka*, 1987, 1999; *Ashkar and Bobée*, 1988; *Ashkar and Ouranda*, 1998]. These approaches are based on normal distribution theory, and exploit the independence of the normal distribution's sample mean and variance. The confidence intervals for quantiles are then expressed in terms of the noncentral T distribution. Some formulas [*Chowdhury and Stedinger*, 1991] employ a correction to compensate for differences between the normal and P-3 distributions. However, applying these results to censored data is problematic for two reasons. First, it is not obvious how to define degrees of freedom when dealing with censored samples. Second, the mean and variance estimators for censored data tend to be highly correlated, which violates one of the fundamental assumptions of normal-based confidence intervals.

4.1. Simple Confidence Intervals

One simple approach for constructing approximate confidence intervals around an unknown parameter requires only the estimated value and its standard deviation [*Kite*, 1988; *Stedinger et al.*, 1993]. The results for a 100ε percent two-sided CI for X_p are

$$\mathrm{CI} \approx (\hat{X}_p + z_{(1-\varepsilon)/2}\hat{\sigma}_{\hat{X}_p}, \hat{X}_p + z_{(1+\varepsilon)/2}\hat{\sigma}_{\hat{X}_p}), \quad (19)$$

where $z_{(1-\varepsilon)/2}$ is the $(1-\varepsilon)/2$ quantile of the standard normal distribution, and the equation for estimating $\sigma_{\hat{X}_p}$ appears as (67) in section A3.

The confidence intervals defined in (19) are simple, but they are not entirely satisfactory because $\hat{X}_p$ is generally highly correlated with its estimated standard deviation, $\hat{\sigma}_{\hat{X}_p}$. Thus, even though

$$Z \equiv \frac{X_p - \hat{X}_p}{\sigma_{\hat{X}_p}} \quad (20)$$

has close to a standard normal distribution [*Cohn et al.*, 1997], this is not the case for the random variable

$$Z_2 \equiv \frac{X_p - \hat{X}_p}{\hat{\sigma}_{\hat{X}_p}}. \quad (21)$$

The consequence of this problem is bias in the confidence interval and failure to provide the nominal coverage, which will be made clear by the Monte Carlo results presented in section 5.3.

4.2. Adjusted Confidence Intervals

The simple confidence intervals in section 4.1 can be modified to correct for the correlation between $\hat{X}_p$ and $\hat{\sigma}_{\hat{X}_p}$. Consider the statistic

$$T \equiv \frac{(X_p - \hat{X}_p)}{\hat{\sigma}_{\hat{X}_p} + \kappa(X_p - \hat{X}_p)}, \quad (22)$$

where

$$\kappa \equiv \frac{\widehat{\mathrm{Cov}}\,[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}]}{\hat{\sigma}^2_{\hat{X}_p}} \quad (23)$$

is a function of the sample size and the censoring threshold (and, to some extent, of α), and is selected so that the numerator and denominator of (22) are asymptotically uncorrelated. Estimators for Cov $[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}]$ and $\sigma^2_{\hat{X}_p}$ are available from (70) in section A4.

The statistic T in (22) can be expressed as a ratio of two random variables

$$T \equiv \frac{Z}{W}, \quad (24)$$

where

$$Z = \frac{(X_p - \hat{X}_p)}{\sigma_{\hat{X}_p}} \quad (25)$$

$$W = \frac{\hat{\sigma}_{\hat{X}_p} + \kappa(X_p - \hat{X}_p)}{\sigma_{\hat{X}_p}} \quad (26)$$

Z, the numerator of (24), is asymptotically standard normal. The denominator W is by construction asymptotically uncorrelated with the numerator and has a mean of 1 and an asymptotic variance of

$$\mathrm{Var}\,[W] = \frac{\sigma^2_{\hat{\sigma}_{\hat{X}_p}} - 2E[\kappa]\,\mathrm{Cov}\,[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}] + E[\kappa^2]\sigma^2_{\hat{X}_p}}{\sigma^2_{\hat{X}_p}}, \quad (27)$$

$$\mathrm{Var}\,[W] = \frac{\sigma^2_{\hat{\sigma}_{\hat{X}_p}} - \dfrac{\mathrm{Cov}^2\,[\hat{X}_p,\ \hat{\sigma}_{\hat{X}_p}]}{\sigma^2_{\hat{X}_p}}}{\sigma^2_{\hat{X}_p}}. \tag{28}$$

Equation (71) provides estimators for the terms appearing on the right-hand side of (28).

The denominator W can be satisfactorily approximated by the square root of a χ^2_ν random variable divided by its mean. The distribution's "degrees of freedom," ν, is defined so as to preserve the second moment of W (the mean of W has already been fixed at 1). This implies setting

$$\nu \equiv \frac{E^2[W]}{2\ \mathrm{Var}\,[W]}, \tag{29}$$

$$\nu = \frac{1}{2\ \mathrm{Var}\,[W]}. \tag{30}$$

(The mean and variance of a χ^2_ν variate, say X, are respectively ν and 2ν; therefore, asymptotically, $\sqrt{(X/\nu)}$ has mean 1 and variance $1/(2\nu)$.) It is important to note that ν is a function of the quantile being estimated, as well as the sample size of the data.

T, expressed as the ratio of a standard normal random variable to the square root of an independent χ^2_ν random variable divided by its degrees of freedom, is recognizable as a Student T_ν variate [*Abramowitz and Stegun*, 1964].

One can rearrange (22) so that X_p appears only on the left-hand side:

$$X_p = \hat{X}_p + \frac{\hat{\sigma}_{\hat{X}_p}T}{1 - \kappa T}. \tag{31}$$

Substituting $T_{\nu,(1-\varepsilon)/2}$ for T in (31) results in adjusted confidence intervals of the form:

$$\left(\hat{X}_p + \frac{\hat{\sigma}_{\hat{X}_p}T_{\nu,(1-\varepsilon)/2}}{1 - \kappa T_{\nu,(1-\varepsilon)/2}},\ \hat{X}_p + \frac{\hat{\sigma}_{\hat{X}_p}T_{\nu,(1+\varepsilon)/2}}{1 - \kappa T_{\nu,(1+\varepsilon)/2}}\right). \tag{32}$$

One caution should be noted: The denominator in the last part of (31), $(1 - \kappa T)$, can be negative for sufficiently large (small, for $\kappa T < 0$) T, which leads to nonsensical confidence intervals. This can occur in small samples ($N_S = 25$) when estimating confidence intervals whose nominal coverage is close to 100% and indicates the limited validity of the approximations employed above. Nonsensical confidence intervals were avoided in the Monte Carlo simulations by constraining the value of κ so that κT was never smaller than 0.5.

4.3. Some Observations Related to the Simple and Adjusted Confidence Intervals

Two types of confidence intervals have been defined, and it is worthwhile to compare some of their properties. Specifically, (1) the simple and the adjusted confidence intervals will always overlap; in particular, they both contain the point estimate for $\hat{X}_p$; and (2) the adjusted intervals around $\hat{X}_p$ will tend to be wider than the corresponding simple intervals, at least for the Pearson type 3 distribution (results for $\hat{Q}_p$ will depend on the parameters). This occurs for two reasons: first, because the T distribution has longer tails than does the normal; and second, even if we approximate T by the normal variate z, the width for the simple intervals is of the form

$$W = (\hat{X}_p + z_{(1+\varepsilon)/2}\hat{\sigma}_{\hat{X}_p}) - (\hat{X}_p + z_{(1-\varepsilon)/2}\hat{\sigma}_{\hat{X}_p}) \tag{33}$$

$$W = 2z_{(1+\varepsilon)/2}\hat{\sigma}_{\hat{X}_p}, \tag{34}$$

whereas the adjusted interval width is of the same form as width = high − low and

$$W = \left(\hat{X}_p + \frac{\hat{\sigma}_{\hat{X}_p}T_{(1+\varepsilon)/2}}{1 - \kappa T_{(1+\varepsilon)/2}}\right) - \left(\hat{X}_p + \frac{\hat{\sigma}_{\hat{X}_p}T_{(1-\varepsilon)/2}}{1 - \kappa T_{(1-\varepsilon)/2}}\right) \tag{35}$$

$$W \approx 2z_{(1+\varepsilon)/2}\hat{\sigma}_{\hat{X}_p}/[1 - (\kappa z_{(1+\varepsilon)/2})^2] \tag{36}$$

Thus the ratio of the adjusted interval width to the simple interval width in large samples is $1/[1 - (\kappa z_{(1+\varepsilon)/2})^2]$. (3) Although the simple intervals are narrower than the adjusted intervals, they are generally not subsets of the adjusted intervals.

5. Monte Carlo Experiments

Monte Carlo experiments were used to address three questions:

1. How well do the asymptotic moment variances (equation (55), Appendix A.1) approximate the true variance of $\hat{\mathbf{M}}$?
2. How well does the linearized quantile variance estimator, $\hat{\sigma}^2_{\hat{X}_p}$ (equation (66), Appendix A3), describe the true quantile variance $\sigma^2_{\hat{X}_p}$?
3. How often do confidence intervals constructed using the methods described in sections 4.1 (equation (19)) using simple confidence intervals, and in 4.2 (equation (32)) using the adjusted confidence intervals, contain the true quantiles?

A number of simulations were run to determine the estimators' performance under a range of circumstances that might be encountered in practice. This included examining the effect of varying the following sampling population parameters.

1. Flood data from the United States seldom exhibits log-skewness outside the range −0.5 to +0.5. Log skews of −0.5, −0.1, +0.1, and +0.5 were used in the experiments. Varying skewness addresses all issues related to the LP-3 parameters because EMA is invariant with respect to location and scale.
2. In all cases it was assumed that the first "class" of flood data consisted of (uncensored) systematic gage data (i.e., $a_1 = -\infty$ and $b_1 = \infty$). Values of N_S of 25 and 100 years were used, which reflect systematic records that might be encountered in practice.
3. In addition to the gage data, a "class" of (historical) flood information was also used. This represents a period of N_H years prior to the systematic record during which large floods—those that exceeded the observation threshold—were recorded. Values of N_H of 0 (no historical information), 50 and 150 years were used. The estimators' performance with longer historical periods, which often arise with paleoflood data [*Jarrett and Malde*, 1987], can be inferred reasonably well from the results with $N_H = 150$.
4. The censoring threshold a_2, the level above which the exact magnitude of floods in the historical period would be recorded, was set at the 90th percentile of the sampling population, $X_{0.90}$.

Each Monte Carlo simulation consisted of 10,000 replicate samples.

5.1. Results for Moments Estimator Variance

Table 1 shows how well asymptotic variances for $\mathbf{M}$ (equation (55), Appendix A1) describe the true moment variance. In the absence of censoring, the asymptotic variances are exact

Table 1. Percentage Differences Between Monte Carlo and Asymptotic Variances and Covariances for First Three Noncentral Moments Given N_S-Year Systematic Record Combined With N_H Years of Historical Information Censored at the 90th Percentile

			Variance-Covariance Elements					
N_S	N_H	Skew	$[\mu_1]$	$[\mu_1, \mu_2]$	$[\mu_1, \mu_3]$	$[\mu_2]$	$[\mu_2, \mu_3]$	$[\mu_3]$
25	0	−0.5	−1	2	−1	−1	−1	−1
		−0.1	−1	7	−2	−2	4	−3
		0.1	−1	7	−2	−2	4	−3
		0.5	−1	2	−1	−1	−1	−1
	50	−0.5	2	−2	2	−5*	−1	9*
		−0.1	1	−6*	0	−3	−2	3
		0.1	0	−3	1	−2	−4	0
		0.5	−2	−24	4	1	10	−4*
	150	−0.5	1	−6*	−1	−8*	0	12*
		−0.1	−1	−9*	−5*	−7*	−7*	0
		0.1	2	0	2	−2	4	7*
		0.5	1	2	2	−2	−13*	−4*
100	0	−0.5	0	−4	−1	2	−2	−2
		−0.1	0	−22	0	0	−8	−1
		0.1	0	−22	0	0	−8	−1
		0.5	0	−4	−1	2	−2	−2
	50	−0.5	0	1	1	0	1	2
		−0.1	1	−10	0	−3	−6	1
		0.1	2	4	2	−1	18	2
		0.5	−1	1	0	0	0	−1
	150	−0.5	1	−3	0	−3	−2	2
		−0.1	0	−5	0	−1	−3	−1
		0.1	0	3	0	1	5	1
		0.5	3*	−174	4*	1	3	0

Presented, as a function of N_S, N_H and population skewness. Asterisks indicate statistical significance at the 5 percent level.

for all sample sizes, so one need consider only cases which include historical information. Even in the small-sample case of $N_S = 25$ and $N_H = 50$, the ratio of the true variances (those observed in Monte Carlo simulation) to the asymptotic variances is close to 1. The most extreme (statistically significant) deviation is around 9%. For the larger sample sizes, only 2 of the 72 variance term differences was statistically significant at the 5% level.

Table 2. Percentage Bias in EMA Quantile Estimator $\hat{X}_p$ and in Estimator for Quantile Standard Deviations $\hat{\sigma}^2_{\hat{X}_p}$

			Quantile Estimated					
			0.90		0.99		0.999	
N_S	N_H	Skew	Bias	SD	Bias	SD	Bias	SD
25	0	−0.5	1.3*	0	−0.2	17*	−0.2	26*
		−0.1	0.3*	2*	0.3*	8*	0.0	12*
		0.1	−0.2*	1	−0.5*	8*	0.0	12*
		0.5	−1.1*	2*	−3.0*	13*	0.2	18*
	50	−0.5	0.9*	−3*	−0.2*	13*	−0.1	23*
		−0.1	0.1*	−2*	0.1*	4*	0.0	8*
		0.1	−0.1*	−1	−0.2*	4*	0.0	7*
		0.5	−0.4*	−1	−1.3*	6*	0.1	10*
	150	−0.5	0.5*	−3*	−0.2*	10*	−0.1	19*
		−0.1	0.1*	−2*	0.1*	3*	0.0	6*
		0.1	−0.1*	0	−0.1*	3*	0.0	4*
		0.5	−0.2*	−1	−0.7*	5*	0.1	7*
100	0	−0.5	0.4*	−2*	−0.1	5*	−0.1	8*
		−0.1	0.1*	−1	0.1*	2*	0.0	3*
		0.1	−0.1*	1	−0.2*	2*	0.0	3*
		0.5	−0.3*	1	−0.8*	4*	0.1	5*
	50	−0.5	0.3*	−1	−0.1	5*	−0.1	8*
		−0.1	0.1*	−1	0.1*	1	0.0	2
		0.1	−0.1*	0	−0.1*	1	0.0	2*
		0.5	−0.2*	0	−0.6*	3*	0.1	4*
	150	−0.5	0.2*	−2*	−0.1*	4*	−0.1	8*
		−0.1	0.0*	−1	0.0*	2*	0.0	4*
		0.1	−0.1*	0	−0.1*	1	0.0	2*
		0.5	−0.1*	−1	−0.4*	2*	0.1	3*

Historical data are censored at the 90% level. Asterisks indicate statistical significance at 5% level.

Table 3. Observed Coverages for Simple and Adjusted Confidence Intervals for $X_{0.99}$ as a Function of Systematic Record Length (N_S), Historical Record Length (N_H), and Population Skewness

			90% Confidence Intervals					
			Simple CI			Adjusted CI		
N_S	N_H	Sk	$\langle X\rangle$	$X\langle\ \rangle$	$\langle\ \rangle X$	$\langle X\rangle$	$X\langle\ \rangle$	$\langle\ \rangle X$
25	0	−0.5	96.8*	0.1*	3.1*	91.9*	3.1*	4.9
		−0.1	90.6*	0.1*	9.3*	89.5	3.5*	7.0*
		0.1	87.4*	0.1*	12.6*	89.2*	3.9*	6.9*
		0.5	83.5*	0.0*	16.5*	90.1	4.1*	5.8*
	50	−0.5	95.7*	0.6*	3.7*	92.0*	3.3*	4.7
		−0.1	90.9*	0.5*	8.6*	90.3	3.8*	5.9*
		0.1	88.4*	0.4*	11.3*	90.3	3.9*	5.8*
		0.5	86.3*	0.1*	13.6*	91.2*	4.3*	4.6*
	150	−0.5	95.4*	1.4*	3.3*	91.7*	3.8*	4.6*
		−0.1	90.6	1.2*	8.2*	90.6	4.4*	5.0
		0.1	89.1*	0.6*	10.3*	90.9*	3.9*	5.3
		0.5	88.0*	0.6*	11.4*	91.2*	4.3*	4.5*
100	0	−0.5	94.3*	1.7*	4.1*	89.9	4.3*	5.9*
		−0.1	90.3	1.3*	8.4*	89.7	4.3*	6.0*
		0.1	89.0*	0.9*	10.2*	89.8	4.3*	5.9*
		0.5	88.5*	0.5*	11.0*	90.8*	4.3*	4.9
	50	−0.5	93.8*	2.0*	4.2*	90.6	4.0*	5.4
		−0.1	90.2	1.5*	8.2*	89.9	4.5*	5.6*
		0.1	89.1*	1.3*	9.6*	90.1	4.1*	5.8*
		0.5	89.1*	0.8*	10.1*	90.5	4.6	4.9
	150	−0.5	93.5*	2.2*	4.3*	90.5	4.3*	5.3
		−0.1	90.8*	2.0*	7.2*	90.4	4.7	4.9
		0.1	89.4	1.5*	9.2*	90.5	3.9*	5.5*
		0.5	89.2*	1.1*	9.7*	90.6	4.6	4.8

Results correspond to a nominal confidence levels of (90%). Three columns correspond to each CI method, ($\langle X\rangle$, $X\langle\ \rangle$, $\langle\ \rangle X$), indicating the percentage of cases in which the estimated confidence interval contained $X_{0.99}$, the estimated CI was greater than $X_{0.99}$, and the estimated CI was less than $X_{0.99}$, respectively. Asterisks indicate statistically significant deviations from expected coverages based on a balanced two-sided test at 5% level.

Table 4. Observed Coverages for Simple and Adjusted Confidence Intervals for $X_{0.99}$ as a Function of Systematic Record Length (N_S), Historical Record Length (N_H), and Population Skewness

			99% Confidence Intervals					
			Simple CI			Adjusted CI		
N_S	N_H	Sk	$\langle X\rangle$	$X\langle\ \rangle$	$\langle\ \rangle X$	$\langle X\rangle$	$X\langle\ \rangle$	$\langle\ \rangle X$
25	0	−0.5	99.7*	0.0*	0.4*	98.1*	0.3*	1.6*
		−0.1	97.8*	0.0*	2.3*	96.8*	0.4	2.8*
		0.1	96.1*	0.0*	3.9*	96.7*	0.6	2.6*
		0.5	93.2*	0.0*	6.8*	97.4*	1.0*	1.7*
	50	−0.5	99.4*	0.0*	0.6	98.8	0.2*	1.0*
		−0.1	97.7*	0.0*	2.4*	97.8*	0.5	1.7*
		0.1	96.8*	0.0*	3.3*	98.2*	0.6	1.1*
		0.5	95.2*	0.0*	4.9*	98.5*	0.8*	0.6
	150	−0.5	99.7*	0.0*	0.3*	98.5*	0.3*	1.2*
		−0.1	98.3*	0.0*	1.8*	98.5*	0.5	1.0*
		0.1	97.3*	0.0*	2.7*	98.8	0.4	0.8*
		0.5	96.3*	0.0*	3.7*	98.9	0.7*	0.4
100	0	−0.5	99.7*	0.0*	0.3*	97.9*	0.4*	1.8*
		−0.1	98.2*	0.0*	1.8*	98.3*	0.5	1.1*
		0.1	97.4*	0.0*	2.6*	98.5*	0.6	0.9*
		0.5	96.6*	0.0*	3.4*	98.8	0.8*	0.4
	50	−0.5	99.5*	0.0*	0.4	98.1*	0.4	1.5*
		−0.1	98.2*	0.0*	1.7*	98.5*	0.6	1.0*
		0.1	97.6*	0.0*	2.4*	98.6*	0.6	0.7*
		0.5	96.9*	0.0*	3.1*	98.7*	0.9*	0.4
	150	−0.5	99.6*	0.1*	0.4*	98.2*	0.4	1.4*
		−0.1	98.5*	0.0*	1.5*	98.6*	0.6	0.8*
		0.1	97.7*	0.0*	2.3*	98.7*	0.5	0.7*
		0.5	97.4*	0.0*	2.6*	98.9	0.7*	0.4

Results correspond to a nominal confidence levels of (99%). Three columns correspond to each CI method, ($\langle X\rangle$, $X\langle\ \rangle$, $\langle\ \rangle X$), indicating the percentage of cases in which the estimated confidence interval contained $X_{0.99}$, the estimated CI was greater than $X_{0.99}$, and the estimated CI was less than $X_{0.99}$, respectively. Asterisks indicate statistically significant deviations from expected coverages based on a balanced two-sided test at 5% level.

Table 5. Observed Coverages for Simple and Adjusted Confidence Intervals for $X_{0.99}$ as a Function of Systematic Record Length (N_S), Historical Record Length (N_H), and Population Skewness

			99.9% Confidence Intervals					
			Simple CI			Adjusted CI		
N_S	N_H	Sk	$\langle X\rangle$	$X\langle\ \rangle$	$\langle\ \rangle X$	$\langle X\rangle$	$X\langle\ \rangle$	$\langle\ \rangle X$
25	0	−0.5	100.0*	0.0*	0.1	99.1*	0.0	0.9*
		−0.1	99.3*	0.0*	0.7*	98.3*	0.1	1.7*
		0.1	98.6*	0.0*	1.4*	98.3*	0.2*	1.5*
		0.5	96.8*	0.0*	3.2*	98.8*	0.3*	0.9*
	50	−0.5	99.9	0.0*	0.1	99.7*	0.0	0.2*
		−0.1	99.2*	0.0*	0.8*	99.4*	0.1	0.6*
		0.1	99.0*	0.0*	1.0*	99.4*	0.1*	0.5*
		0.5	97.9*	0.0*	2.1*	99.6*	0.2*	0.2*
	150	−0.5	100.0*	0.0*	0.0	99.6*	0.0*	0.4*
		−0.1	99.5*	0.0*	0.5*	99.6*	0.1	0.4*
		0.1	99.1*	0.0*	0.9*	99.6*	0.1	0.3*
		0.5	98.7*	0.0*	1.3*	99.7*	0.2*	0.1
100	0	−0.5	100.0*	0.0*	0.0*	99.3*	0.0	0.7*
		−0.1	99.6*	0.0*	0.4*	99.4*	0.1	0.5*
		0.1	99.2*	0.0*	0.8*	99.6*	0.1	0.3*
		0.5	98.7*	0.0*	1.3*	99.8*	0.2*	0.1
	50	−0.5	100.0*	0.0*	0.0	99.4*	0.0*	0.6*
		−0.1	99.6*	0.0*	0.4*	99.6*	0.1	0.3*
		0.1	99.3*	0.0*	0.7*	99.7*	0.2*	0.1*
		0.5	99.0*	0.0*	1.0*	99.8	0.1*	0.0
	150	−0.5	100.0*	0.0*	0.1	99.5*	0.1	0.4*
		−0.1	99.7*	0.0*	0.3*	99.7*	0.1	0.2*
		0.1	99.2*	0.0*	0.8*	99.7*	0.1	0.2*
		0.5	99.1*	0.0*	0.9*	99.7*	0.2*	0.1

Results correspond to a nominal confidence levels of (99.9%). Three columns correspond to each CI method, ($\langle X\rangle$, $X\langle\ \rangle$, $\langle\ \rangle X$), indicating the percentage of cases in which the estimated confidence interval contained $X_{0.99}$, the estimated CI was greater than $X_{0.99}$, and the estimated CI was less than $X_{0.99}$, respectively. Asterisks indicate statistically significant deviations from expected coverages based on a balanced two-sided test at 5% level.

5.2. Results for the Quantile Estimator

One criterion for judging the performance of an estimator is bias, which is the tendency of an estimator to over- or underestimate the parameter of interest. The "bias" columns in Table 2 show the bias of $\hat{X}_{0.99}$ (assuming $\tau = 0$). The bias did not exceed 1.3% for the cases tested and declined rapidly with larger sample sizes, confirming the finding of *Cohn et al.* [1997] that the EMA estimator is not substantially biased.

The "SD" columns in Table 2 show how well the quantile standard deviation estimator given in (65), $\hat{\sigma}_{\hat{X}_p}$, describes the true quantile standard deviation, $\sigma_{\hat{X}_p}$. Even for the small-sample case, the asymptotic estimates are always within 17% for $\sigma_{\hat{X}_{0.99}}$. The estimated standard deviations are always within 5% for the large sample cases. This suggests that the linear approximation used to derive the variances is reasonably good.

5.3. Results for Confidence Intervals

Tables 3, 4, and 5 show how often the simple and adjusted confidence intervals actually contain the true quantiles and the percentage of "failures" that occurred in the right and left tails. Figures 2 and 3 present the same results for the cases corresponding to $N_S = 25$ and $N_H = 50$.

The coverages for the simple confidence intervals are roughly correct, though far from exact with small samples. For a nominal 90% confidence interval, the overall coverage ranged from 83.5 to 96.8%. As can be seen in Figure 2, nearly all of the failures occurred when the confidence interval was below the true quantile. This is because $\hat{X}_p$ is positively correlated with $\hat{\sigma}_{\hat{X}_p}$. When the confidence interval (whose center is $\hat{X}_p$) is low, the confidence interval (whose width is proportional to $\hat{\sigma}_{\hat{X}_p}$) is likely to be too narrow; when it is too high it is likely to be too wide.

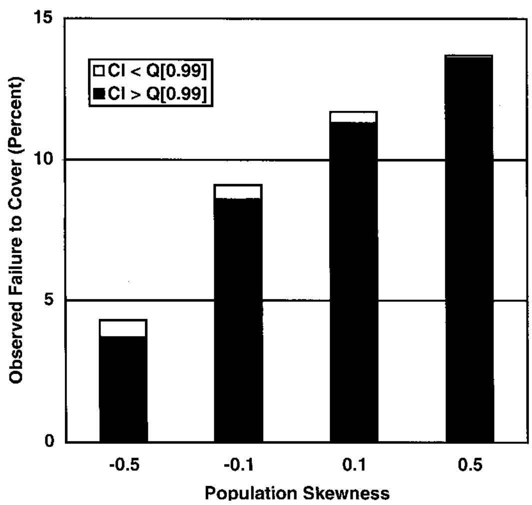

Figure 2. The frequency with which the simple 90% confidence intervals fail to cover the true quantile (in this case the 99th percentile of the frequency distribution), as a function of population skewness. The two sections of each bar indicate the frequency with which a failure occurred because the confidence interval was above or below the true quantile. Results are based on Monte Carlo simulation with a systematic record length of $N_S = 25$, a historical period of length $N_H = 50$, and a censoring threshold at the 90th percentile.

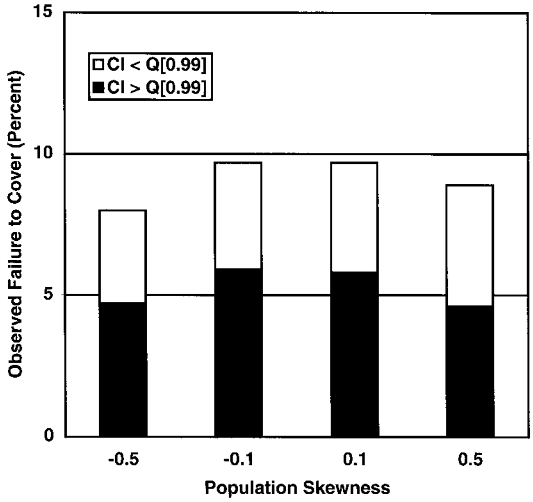

Figure 3. The frequency with which the adjusted 90% confidence intervals fail to cover the true quantile (in this case the 99th percentile of the frequency distribution), as a function of population skewness. The two sections of each bar indicate the frequency with which a failure occurred because the confidence interval was above or below the true quantile. Results are based on Monte Carlo simulation with a systematic record length of $N_S = 25$, a historical period of length $N_H = 50$, and a censoring threshold at the 90th percentile.

The results for the adjusted confidence intervals are considerably better. The nominal 90% confidence intervals provided coverages between 89.2 and 92.0% in all cases. Not only were overall coverages close to nominal values, but, as can be seen in Figure 3, the failures occur with nearly equal frequency in the left and right tail. For the 99.9% confidence intervals the adjusted confidence intervals were found to be consistently too narrow, although they still performed substantially better than the simple confidence intervals.

5.4. Discussion of Results

The results in section 5.3 suggest that confidence intervals constructed according to the adjusted confidence interval procedure, though not exact, achieved nearly the specified coverages. *Cohn et al.* [1997] had previously noted that even with modest-sized samples, the EMA quantile estimator was nearly normally distributed, nearly unbiased, and its variance was close to the Frechet-Cramér-Rao information bounds [*Rohatgi*, 1976; *Kendall et al.*, 1979, chapter 17.15, p. 9]. Thus it is not surprising that asymptotic theory accurately predicts the behavior of EMA quantile estimators.

6. Concerns About Historical and Paleoflood Information

This paper provides a method for computing the uncertainty in EMA flood quantile estimates. This should facilitate increased use of historical and paleoflood information in flood studies. One must be careful, however, because the value of historical information depends on several hard to test assumptions: (1) That sampling characteristics and accuracy of historical data are understood; (2) that the data are representative of the target population; (3) that historical and paleoflood information is correctly interpreted; (4) that an adequate statistical flood model is employed; (5) that appropriate statistical estimation methods are used; (6) that one can specify the values for a and b; (7) and that one can confidently assign every historical or paleo flood to the correct category (W. Kirby, oral communication, U.S. Geological Survey, 2000).

Similar assumptions are needed, and must be justified, when using systematic data alone, but we have more experience working with such data and have acquired confidence in our methods. As researchers gain experience with historical data sources and develop better methods for interpreting the data and as practitioners make increased use of this information in their flood frequency studies, it seems likely that many of the concerns about historical information will be resolved.

In its recent study of California's American River [*Potter*, 1999] a National Research Council Committee felt justified in recommending use of the 57 years of at-site historical information to augment the 92-year systematic record. The Committee chose not to employ a paleoflood record, extending back 3000 to 5000 years, because of concerns about some of the assumptions listed above.

Nonetheless, there remains need for additional research on historical flood information and its interpretation. There are also issues related to EMA that need further research, including (1) how best to incorporate regional information; (2) how to incorporate measurement error; and (3) how, exactly, to implement each of the components of Bulletin 17B in the context of EMA.

7. Conclusions

This paper shows how first-order methods can be used to derive approximations for the variance of EMA moments and flood quantile estimators. Approximate confidence intervals are developed using normal theory with and without an adjustment to correct for the correlation between the quantile estimate and its estimated standard deviation. Monte Carlo experiments show that the adjusted confidence intervals for flood quantiles, while not exact, provide reasonably accurate descriptions of the uncertainty in flood quantile estimates.

Appendix A

A1. Derivation of EMA Moments Variance

This section outlines a derivation of the asymptotic variance, $\hat{\Sigma}$, of the EMA moments estimator $\hat{\mathbf{M}}$. The derivation involves power-series expansions of the nonlinear terms in (8) and then discarding low-order terms.

Equation (8) defines the EMA estimate $\hat{\mathbf{M}}$ as the value of $\mathbf{M}$ that satisfies the matrix equation:

$$\mathbf{M} = (1/N) \sum_{i=1}^{N} \boldsymbol{\chi}(\psi(X_i), \mathbf{M}) \mathcal{I}[\psi(X_i)]. \tag{37}$$

One can use (37) to derive a first-order approximation to $\hat{\Sigma}$ without explicitly solving the equation. Subtracting $\boldsymbol{\mu}_{\mathbf{M}} = E[\mathbf{M}]$ from both sides of (8) and multiplying by N yields

$$N(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) = \sum_{i=1}^{N} \boldsymbol{\chi}(\psi(X_i), \mathbf{M}) \mathcal{I}[\psi(X_i)] - N\boldsymbol{\mu}_{\mathbf{M}}. \tag{38}$$

Define

$$N_L \equiv \sum_{i=1}^{N} \mathcal{I}(X_i < a) \tag{39}$$

$$N_G \equiv \sum_{i=1}^{N} \mathscr{I}(X_i > b). \tag{40}$$

Thus

$$\sum_{i=1}^{N} \boldsymbol{\chi}(\psi(X_i), \mathbf{M})\mathscr{I}[\psi(X_i)] = N_L E_{\boldsymbol{\theta}[\mathbf{M}]}[\mathbf{X}|X < a] + N_G E_{\boldsymbol{\theta}[\mathbf{M}]}[\mathbf{X}|X > b] + \sum_{i=1}^{N_B} \mathbf{X}_i, \tag{41}$$

where the final summation is taken over $X \varepsilon [a, b]$. The expected values, $E_{\boldsymbol{\theta}[\mathbf{M}]}[\mathbf{X}|X < a]$, can be expanded in a series in $(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}})$:

$$E_{\boldsymbol{\theta}[\mathbf{M}]}[\mathbf{X}|X < a] \approx \boldsymbol{\mu}_{\mathbf{X}_L} + \mathbf{J}_L(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}), \tag{42}$$

where

$$\boldsymbol{\mu}_{\mathbf{X}_L} = E_{\boldsymbol{\theta}[\boldsymbol{\mu}_\mathbf{M}]}[\mathbf{X}|X < a] \tag{43}$$

$\mathbf{J_L}$

$$= \begin{bmatrix} \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X|X<a]}{\partial \mathbf{m}_1} & \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X|X<a]}{\partial \mathbf{m}_2} & \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X|X<a]}{\partial \mathbf{m}_3} \\ \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X^2|X<a]}{\partial \mathbf{m}_1} & \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X^2|X<a]}{\partial \mathbf{m}_2} & \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X^2|X<a]}{\partial \mathbf{m}_3} \\ \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X^3|X<a]}{\partial \mathbf{m}_1} & \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X^3|X<a]}{\partial \mathbf{m}_2} & \frac{\partial E_{\boldsymbol{\theta}[\mathbf{M}]}[X^3|X<a]}{\partial \mathbf{m}_3} \end{bmatrix}. \tag{44}$$

Analogously, $E_{\boldsymbol{\theta}[\mathbf{M}]}[\mathbf{X}|X > b]$ can be expanded around $\boldsymbol{\mu}_{\mathbf{X}_G}$ with Jacobian $\mathbf{J_G}$. Equation (41) can then be linearized in terms of $(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}})$ to give

$$\sum_{i=1}^{N} \boldsymbol{\chi}(\psi(X_i), \mathbf{M})\mathscr{I}[\psi(X_i)] \approx N_L\boldsymbol{\mu}_{\mathbf{X}_L} + N_G\boldsymbol{\mu}_{\mathbf{X}_G} + (N_L\mathbf{J}_L + N_G\mathbf{J}_G)(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) + \sum_{i=1}^{N_B} \mathbf{X}_i. \tag{45}$$

Equation (45) can be further expanded about the mean of N_L and N_G and low-order terms discarded

$$\begin{aligned} \sum_{i=1}^{N} \boldsymbol{\chi}(\psi(X_i), \mathbf{M})\mathscr{I}[\psi(X_i)] &\approx N_L\boldsymbol{\mu}_{\mathbf{X}_L} + N_G\boldsymbol{\mu}_{\mathbf{X}_G} \\ &+ (\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G)(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) + [(N_L - \mu_{N_L})\mathbf{J}_L \\ &+ (N_G - \mu_{N_G})\mathbf{J}_G](\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) + \sum_{i=1}^{N_B} \mathbf{X}_i \approx N_L\boldsymbol{\mu}_{\mathbf{X}_L} + N_G\boldsymbol{\mu}_{\mathbf{X}_G} \\ &+ (\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G)(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) + \sum_{i=1}^{N_B} \mathbf{X}_i, \end{aligned} \tag{46}$$

where $\mu_{N_L} \equiv E[N_L]$ and $\mu_{N_G} \equiv E[N_G]$ (and analogously $\mu_{N_B} \equiv E[N_B]$). Substituting (46) into (38) results in a linear equation in $(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}})$:

$$N(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) = N_L\boldsymbol{\mu}_{\mathbf{X}_L} + N_G\boldsymbol{\mu}_{\mathbf{X}_G} + (\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G) \cdot (\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) + \sum_{i=1}^{N_B} \mathbf{X}_i - N\boldsymbol{\mu}_{\mathbf{M}}, \tag{47}$$

which can be rearranged to yield

$$N\left(\mathbf{I} - \frac{\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G}{N}\right)(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) = N_L\boldsymbol{\mu}_{\mathbf{X}_L} + N_G\boldsymbol{\mu}_{\mathbf{X}_G} + \sum_{i=1}^{N_B} \mathbf{X}_i - N\boldsymbol{\mu}_{\mathbf{M}}. \tag{48}$$

Equation (48) can be solved for $(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}})$:

$$\begin{aligned} (\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}) &= (1/N)\left(\mathbf{I} - \frac{\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G}{N}\right)^{-1} \cdot \left(N_L\boldsymbol{\mu}_{\mathbf{X}_L} + N_G\boldsymbol{\mu}_{\mathbf{X}_G} + \sum_{i=1}^{N_B} \mathbf{X}_i - N\boldsymbol{\mu}_{\mathbf{M}}\right) \\ &\approx \left(\mathbf{I} - \frac{\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G}{N}\right)^{-1} \cdot \left[\boldsymbol{\mu}_{\mathbf{X}}\mathbf{N} + \sum_{i=1}^{\mu_{N_B}} (\mathbf{X}_i - \boldsymbol{\mu}_{\mathbf{X}_B}) - N\boldsymbol{\mu}_{\mathbf{M}}\right], \end{aligned} \tag{49}$$

where

$$\boldsymbol{\mu}_{\mathbf{X}} \equiv [\boldsymbol{\mu}_{\mathbf{X}_L} \quad \boldsymbol{\mu}_{\mathbf{X}_B} \quad \boldsymbol{\mu}_{\mathbf{X}_G}] \tag{50}$$

$$\boldsymbol{\mu}_{\mathbf{X}} \equiv \begin{bmatrix} E_{\boldsymbol{\theta}}[X|X<a] & E_{\boldsymbol{\theta}}[X|a \le X \le b] & E_{\boldsymbol{\theta}}[X|X>b] \\ E_{\boldsymbol{\theta}}[X^2|X<a] & E_{\boldsymbol{\theta}}[X^2|a \le X \le b] & E_{\boldsymbol{\theta}}[X^2|X>b] \\ E_{\boldsymbol{\theta}}[X^3|X<a] & E_{\boldsymbol{\theta}}[X^3|a \le X \le b] & E_{\boldsymbol{\theta}}[X^3|X>b] \end{bmatrix} \tag{51}$$

$$\mathbf{N} \equiv \begin{bmatrix} N_L \\ N_B \\ N_G \end{bmatrix}. \tag{52}$$

Substituting

$$\begin{aligned} \mathbf{B} &= \boldsymbol{\mu}_{\mathbf{X}}\mathbf{N} \\ \mathbf{C} &= \sum_{i=1}^{\mu_{N_B}} (\mathbf{X}_i - \boldsymbol{\mu}_{\mathbf{X}_B}) \\ \mathbf{D} &= \frac{\mu_{N_L}\mathbf{J}_L + \mu_{N_G}\mathbf{J}_G}{N} \\ \mathbf{A} &= (\mathbf{I} - \mathbf{D})^{-1} \end{aligned} \tag{53}$$

into (49) and adding $N\boldsymbol{\mu}_{\mathbf{M}}$ to both sides yields

$$N\mathbf{M} = \mathbf{A}(\mathbf{B} + \mathbf{C}) + \text{const.} \tag{54}$$

Dividing both sides of (54) by N, discarding the constant, and taking variances of both sides yields the variance of $\mathbf{M}$:

$$\hat{\Sigma} = \frac{1}{N^2}\mathbf{A}(\text{Var}[\mathbf{B}] + \text{Var}[\mathbf{C}])\mathbf{A}'. \tag{55}$$

Note that all of the elements in $\mathbf{A}$ converge rapidly to constants, and $\mathbf{B}$ is uncorrelated with $\mathbf{C}$ because the value of each X_i in the interval $[a, b]$ is independent of the number of X_i in the interval.

The variance of $\mathbf{B}$ is given by

$$\text{Var}[\mathbf{B}] = \boldsymbol{\mu}_{\mathbf{X}} \text{Var}[\mathbf{N}]\boldsymbol{\mu}_{\mathbf{X}}'. \tag{56}$$

N has a multinomial distribution [*DeGroot*, 1975, p. 247] with variance

$$\mathrm{Var}\,[\mathbf{N}] = N\begin{bmatrix} p_L(1-p_L) & -p_Lp_B & -p_Lp_G \\ -p_Lp_B & p_B(1-p_B) & -p_Bp_G \\ -p_Lp_G & -p_Bp_G & p_G(1-p_G) \end{bmatrix}, \tag{57}$$

where

$$\begin{bmatrix} p_L \\ p_B \\ p_G \end{bmatrix} = \begin{bmatrix} P[X < a] \\ P[a \le X \le b] \\ P[X > b] \end{bmatrix}. \tag{58}$$

The large-sample variance of **C** is the expected value of the number of terms multiplied by the variance of each term:

$$\mathrm{Var}\,[\mathbf{C}] = \mu_{N_B}\begin{bmatrix} V_{1,1} & V_{1,2} & V_{1,3} \\ V_{2,1} & V_{2,2} & V_{2,3} \\ V_{3,1} & V_{3,2} & V_{3,3} \end{bmatrix}, \tag{59}$$

where

$$V_{i,j} \equiv E_{\boldsymbol{\theta}}[X^{i+j}|a \le X \le b] - E_{\boldsymbol{\theta}}[X^i|a \le X \le b]E_{\boldsymbol{\theta}}[X^j|a \le X \le b] \tag{60}$$

and the expected values, $E_{\boldsymbol{\theta}}[X^p|a \le X \le b]$ are available in (15).

A2. A Special Case: Variance of Uncensored Moments Estimates

In the absence of censoring, the exact variance of $\hat{\mathbf{M}}$ can be computed directly from (55) in Appendix A1. N is the systematic record length. Substituting $a = -\infty$ and $b = \infty$, we find that $\mathbf{J} = [\mathbf{0}]$, Var $[\mathbf{N}] = [\mathbf{0}]$, and only the middle column of $\boldsymbol{\mu}_{\mathbf{X}}$—$[E[X], E[X^2], E[X^3]]'$—needs to be considered. Thus $\mathbf{A} = \mathbf{I}$, $\mathbf{B} = 0$, and (55) reduces to

$$\tilde{\Sigma} = (1/N)\begin{bmatrix} V_{1,1} & V_{1,2} & V_{1,3} \\ V_{2,1} & V_{2,2} & V_{2,3} \\ V_{3,1} & V_{3,2} & V_{3,3} \end{bmatrix}, \tag{61}$$

where

$$V_{i,j} \equiv E_{\boldsymbol{\theta}}[X^{i+j}] - E_{\boldsymbol{\theta}}[X^i]E_{\boldsymbol{\theta}}[X^j]. \tag{62}$$

Equation (3) defines the expectations, $E_{\boldsymbol{\theta}}[X^k]$.

A3. Derivation of EMA Quantile Variance

The asymptotic variance of $\hat{X}_p$ can be obtained from a first-order expansion of $\hat{X}_p$ as a function of **M**:

$$\hat{X}_p \approx X_p + \mathbf{J}_{\hat{X}_p}(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}), \tag{63}$$

where

$$\mathbf{J}_{\hat{X}_p} = \begin{bmatrix} \dfrac{\partial \hat{X}_p}{\partial \hat{m}_1} & \dfrac{\partial \hat{X}_p}{\partial \hat{m}_2} & \dfrac{\partial \hat{X}_p}{\partial \hat{m}_3} \end{bmatrix}. \tag{64}$$

The Jacobian can be evaluated by first computing derivatives with respect to $\{\alpha, \beta, \tau\}$ and then applying the chain rule.

The variance of $\hat{X}_p$ can be approximated:

$$\tilde{\sigma}^2_{\hat{X}_p} \approx \mathbf{J}_{\hat{X}_p} \cdot \tilde{\Sigma} \cdot \mathbf{J}'_{\hat{X}_p}, \tag{65}$$

where the linearized standard deviation $\tilde{\sigma}_{\hat{X}_p}$ is defined as $(\tilde{\sigma}^2_{\hat{X}_p})^{1/2}$.

One can also define an estimator $\hat{\sigma}_{\hat{X}_p}$ for the standard deviation of $\hat{X}_p$, by employing (65) with estimated parameters. The function $\hat{\sigma}_{\hat{X}_p}$ is a random function of $\hat{\mathbf{M}}$. Employing the same approach as before, one can linearize $\hat{\sigma}_{\hat{X}_p}$ into

$$\hat{\sigma}_{\hat{X}_p} \approx \sigma_{\hat{X}_p} + \mathbf{J}_{\hat{\sigma}_{\hat{X}_p}}(\mathbf{M} - \boldsymbol{\mu}_{\mathbf{M}}), \tag{66}$$

where

$$\mathbf{J}_{\hat{\sigma}_{\hat{X}_p}} = \begin{bmatrix} \dfrac{\partial \hat{\sigma}_{\hat{X}_p}}{\partial \hat{m}_1} & \dfrac{\partial \hat{\sigma}_{\hat{X}_p}}{\partial \hat{m}_2} & \dfrac{\partial \hat{\sigma}_{\hat{X}_p}}{\partial \hat{m}_3} \end{bmatrix}. \tag{67}$$

Some of the equations (here and elsewhere in the paper) involve fairly hard to evaluate derivatives. Although algebraic results were used for most parts of this paper, the Jacobian in (67) was evaluated by using numerical differentiation (specifically the "DIFF" subroutine, written by David Kahaner and available on the Web at http://gams.nist.gov/).

A4. Variance-Covariance of $\hat{X}_p$ and $\hat{\sigma}_{\hat{X}_p}$

Equations (64) and (67) can be used to approximate the variance-covariance matrix of $\hat{X}_p$ and $\hat{\sigma}_{\hat{X}_p}$:

$$\mathrm{Var}\begin{bmatrix} \hat{X}_p \\ \hat{\sigma}_{\hat{X}_p} \end{bmatrix} \equiv \begin{bmatrix} \hat{\sigma}^2_{\hat{X}_p} & \mathrm{Cov}\,[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}] \\ \mathrm{Cov}\,[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}] & \hat{\sigma}^2_{\hat{X}_p} \end{bmatrix}, \tag{68}$$

$$\mathrm{Var}\begin{bmatrix} \hat{X}_p \\ \hat{\sigma}_{\hat{X}_p} \end{bmatrix} \approx \begin{bmatrix} \mathbf{J}_{\hat{X}_p} \\ \mathbf{J}_{\hat{\sigma}_{\hat{X}_p}} \end{bmatrix} \tilde{\Sigma} [\mathbf{J}'_{\hat{X}_p} \quad \mathbf{J}'_{\hat{\sigma}_{\hat{X}_p}}], \tag{69}$$

$$\mathrm{Var}\begin{bmatrix} \hat{X}_p \\ \hat{\sigma}_{\hat{X}_p} \end{bmatrix} \equiv \begin{bmatrix} \tilde{\sigma}_{\hat{X}_p} & \widetilde{\mathrm{Cov}}\,[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}] \\ \widetilde{\mathrm{Cov}}\,[\hat{X}_p, \hat{\sigma}_{\hat{X}_p}] & \hat{\sigma}^2_{\hat{X}_p} \end{bmatrix}. \tag{70}$$

If one substitutes $\hat{\Sigma}$ for $\tilde{\Sigma}$, one obtains an estimator for the variance-covariance matrix of $\hat{X}_p$ and $\hat{\sigma}_{\hat{X}_p}$.

A5. Avoiding Multicollinearity of the Moments

Noncentral moments prove to be convenient with respect to algebra, but they create potential numerical problems with respect to computation. The moments are highly collinear, which makes the methods highly sensitive to errors; the finite precision of most programming languages may not be adequate for computing acceptable results.

A general solution to multicollinearity problems is to orthogonalize the variables with orthogonal polynomials [*Press et al.*, 1986]. A simple and often effective alternative is to "center" the variable by fixing the population parameter τ at $\tau = -\alpha\beta$. This sufficiently reduces the multicollinearity of the noncentral moments to eliminate the numerical problems. Note that it makes no difference what value of τ is used because the variance of EMA moments and quantile estimators is invariant to τ.

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T. A. Cohn, U.S. Geological Survey, MS-107, Reston, VA 20192.
W. L. Lane, 1091 Xenophon Street, Golden, CO 80401.
J. R. Stedinger, Cornell University, Ithaca, NY 14853.

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